

# Gulf of Alaska flathead sole stock assessment

By

Benjamin J. Turnock, Thomas K. Wilderbuer and Eric S. Brown

## Summary

A model for flathead sole in the Gulf of Alaska was developed for predicting abundance and ABC levels. Analysis of maturity by age and length was completed for Gulf of Alaska flathead sole in 2002 (J. Stark, pers. comm.) and included in this assessment for estimation of reference fishing mortality values.

The 2003 survey biomass and length data were added to the model. The catch for 2003 was updated and the length data and catch for 2003 was added to the model. The 2003 survey biomass increased to 258,609 t from 170,915 t in 2001.

ABC for 2004 using  $F_{40\%} = 0.47$  was estimated at 51,721 t. Recent catches have been about 2,000 t. ABC for 2003 using  $F_{40\%} = 0.417$  was estimated at 41,402 t (Turnock, et al. 2002). The model estimates of age 3+ biomass increased from about 256,600 t in 1984 to about 298,900 t in 1996, decreased to about 287,000 t in 2000, then increased to 291,400 t in 2003.

## Introduction

A model for Gulf of Alaska flathead sole was developed for predicting abundance and ABC levels. Flathead sole occur from Northern California to the Bering Sea, and possibly the Okhotsk Sea to Japan. Catch has been at a relatively low level, increasing to a high of about 3,100 t in 1996, declining to 900 t in 1999 then increasing to 1,969 t in 2003 (through 2 October 2003) (Table 6.1). In recent years 80% to 90% of the catch has been retained.

Survey abundance estimates were higher in 1984, 1990 and 2003 than in other years, however the trend appears relatively flat (Table 6.2). Survey biomass was 258,609 t in 2003, an increase from the 2001 survey biomass of 170,915 t. The 2001 survey did not cover the eastern Gulf of Alaska. The 2001 survey biomass without the estimated eastern Gulf portion was 153,747 t. The average biomass estimated for the 1993 to 1999 surveys was used to estimate the biomass in the eastern Gulf for 2001. The eastern gulf biomass has been between 8% and 9% of the total biomass for the 1993-1999 surveys.

## Analytic approach

### Model Structure

The model structure was developed following Fournier and Archibald's (1982) methods, with many similarities to Methot (1990). We implemented the model using automatic differentiation software developed as a set of libraries under C++ (ADModel Builder). ADModel Builder can estimate a large number of parameters in a non-linear model using automatic differentiation software extended from Greiwanck and Corliss (1991) and developed into C++ class libraries. This software provides the derivative calculations needed for finding the objective function via a quasi-Newton function minimization routine (e.g., Press et al. 1992). The model implementation language (ADModel Builder) gives simple and rapid access to these routines and provides the ability to estimate the variance-covariance matrix for all parameters of interest. Details of the population dynamics and estimation equations, description of variables and likelihood equations are presented in Appendix A (Tables A.1, A.2

and A.3). There were a total of 68 parameters estimated in the model (Table A.4). The instantaneous natural mortality rate, catchability for the survey and the von Bertalanffy growth parameters were fixed in the model (Table A.5).

### Model assumptions

All emphasis values were 1 except for the catch likelihood where a high emphasis was used to fit the catch closely, while allowing some deviation from the observed catch. The model estimates size compositions using a fixed length-age transition matrix estimated from the 1984, 1993 and 1996 survey length and age data combined. The distribution of lengths within ages was assumed to be normal with coefficients of variation (cv's) estimated from the length at age data of 0.10 for age 3 and 0.08 for age 20.

### Data Sources

The model simulates the dynamics of the population and compares the expected values of the population characteristics to those observed from surveys and fishery sampling programs.

The following data sources were used in the model:

| Data component   | Years                                   |
|--|---|
| Fishery catch (Table 6.1)                                | 1978-2003                               |
| NMFS triennial trawl survey biomass and S.E. (Table 6.2) | 1984,1987,1990,1993,1996,1999,2001,2003 |
| Fishery size compositions                                | 1985-2003                               |
| NMFS triennial trawl survey size compositions            | 1987,1990,1999,2001,2003                |
| NMFS triennial trawl survey age composition data         | 1984,1993,1996.                         |

### Parameters estimated independently

#### *Natural mortality*

Natural mortality was fixed at 0.2. Age at recruitment was set at three in the model due to the small number of fish caught at younger ages. The maximum age of flathead sole based on otolith age determinations is estimated at 25 years.

#### *Growth*

The growth parameter  $L_{inf}$  was estimated at 44.37 cm for females and 37.36 cm for males (Figure 6.1). The length at age 2 ( $L_2$ ) was estimated at 10.17 cm for males and 13.25 cm for females. The growth parameter  $k$  was estimated at 0.157 for females and 0.204 for males. Length at age  $t$  was modeled as:

$$L_t = L_{inf} + (L_2 - L_{inf}) * \exp(-k(t - 2)).$$

#### *Weight at length*

The weight-length relationship for flathead sole is,  $W = 0.00428 L^{3.2298}$ , for both sexes combined where weight is in grams and length in centimeters. Weight at age was estimated using the mean length at age and the weight-length relationship.

### *Maturity*

Gulf of Alaska flathead sole maturity was estimated using histological analysis of ovaries collected in a January 1999 study (J. Stark, pers. comm.). A total of 180 samples were analyzed for estimation of age at maturity. Size at 50% mature was estimated to be 32.0 cm with a slope of 0.775 from a sample of 208 fish. Age at 50% mature was 7.9 with a slope of 1.115. Size at 50% mature was also estimated at 32.0 cm for Bering sea flathead sole, however, age at 50% mature was 9.7 due to slower growth in the Bering sea.

### **Parameters estimated conditionally**

There were a total of 68 parameters estimated in the model (Table A.4). These consist primarily of parameters on the recruitment of flathead sole to the population (and the initial age composition) and values related to annual fishing mortality. The separable age-component of fishing mortality was modeled using a two parameter ascending logistic function estimated separately for males and females. The same form of curve was also used for estimating relative survey age-specific catchability.

## **Results**

Selectivity estimates show that the fishery generally catches flathead sole at older ages than the survey (Fig. 6.2). Fits to the size composition data from the fishery are shown in Figure 6.3 for females and Figure 6.4 for males. The fit to the survey size composition data are in Figure 6.5 for females and Figure 6.6 for males. The survey age composition data are shown in Figures 6.7 and 6.8. On balance, the model estimates fit the length observations reasonably well. The observed age data do not always show consistent year class strengths between males and females. The model estimates of age composition generally fit well except for where there are differences between male and female observed age compositions.

### *Model estimates of biomass*

The model estimates of age 3+ biomass increased from about 255,600 t in 1984 to about 298,900 t in 1996, decreased to 287,000 t in 2000, then increased to 291,400 t in 2003 (Table 6.3 and Fig. 6.9). The fit to the survey biomass estimates is shown in Figure 6.10. The model estimates an increasing biomass despite the high observed biomass in 1984 and lower biomass in 1987 due to the survey length and age data, which indicate a relatively large cohort moving through the population during that time period. The model estimates a slightly higher biomass in 2003 compared to 2001 due to the higher 2003 survey biomass estimate. The model estimates from the 2002 assessment declined from 2001 to 2003.

### *Model estimates of recruitment*

The model estimates of age 3 recruits were lower than average for 1996 to 1999, then higher than average for 2000 to 2002 to attempt to fit the higher 2003 survey biomass estimate (Table 6.3 and Fig. 6.11). The estimated annual recruitment variability was quite low ( $cv = 0.26$ ) and the uncertainty in the actual year-class strengths for recent recruitments was quite high (CV between about 0.4 and 0.7). This is presumably due to the fact that length composition data (the main source of information on year-class variability in recent years) fails to resolve distinct year-classes.

### *Spawner-Recruit Relationship*

No spawner-recruit curve was used in the Model. Recruitments were estimated as deviations from a mean value on a log scale with a modest penalty on outliers (Table A.2).

## Reference fishing mortality rates and yields

Reliable estimates of biomass,  $B_{35\%}$ ,  $F_{35\%}$  and  $F_{40\%}$ , are available, and current biomass is greater than  $B_{40\%}$ . Therefore, flathead sole is in tier 3a of the ABC and overfishing definitions. Under this definition,  $F_{\text{off}} = F_{35\%}$ , and  $F_{\text{ABC}}$  is less than or equal to  $F_{40\%}$ .

Yield for 2004 using  $F_{40\%} = 0.47$  was estimated at 51,721 t. Yield at  $F_{35\%} = 0.63$  was estimated at 64,750 t. Fishing mortality values are relatively high because the age at 50% selected in the fishery is about 9.5 years, while the age at 50% mature is about 8 years. The fishery selectivities reach 95% at about age 13 for females. The  $F$  at age 10 is about 0.25.

## Maximum sustainable yield

Since there is no estimate of the spawner-recruit relationship for flathead sole, no attempt has been made to estimate MSY. However, using the projection model described in the next section, female spawning biomass with  $F=0$  was estimated at 119,240 t.  $B_{35\%}$  (equilibrium female spawning biomass with fishing at  $F_{35\%}$ ) is estimated at 41,734 t.

## Projected catch and abundance

A standard set of projections is required for each stock managed under Tiers 1, 2, or 3 of Amendment 56. This set of projections encompasses seven harvest scenarios designed to satisfy the requirements of Amendment 56, the National Environmental Protection Act, and the Magnuson-Stevens Fishery Conservation and Management Act (MSFCMA).

For each scenario, the projections begin with the vector of 2003 numbers at age estimated in the assessment. This vector is then projected forward to the beginning of 2004 using the schedules of natural mortality and selectivity described in the assessment and the best available estimate of total (year-end) catch for 2003. In each subsequent year, the fishing mortality rate is prescribed on the basis of the spawning biomass in that year and the respective harvest scenario. In each year, recruitment is drawn from an inverse Gaussian distribution whose parameters consist of maximum likelihood estimates determined from recruitments estimated in the assessment. Spawning biomass is computed in each year based on the time of peak spawning and the maturity and weight schedules described in the assessment. Total catch is assumed to equal the catch associated with the respective harvest scenario in all years. This projection scheme is run 1000 times to obtain distributions of possible future stock sizes, fishing mortality rates, and catches.

Five of the seven standard scenarios will be used in an Environmental Assessment prepared in conjunction with the final SAFE. These five scenarios, which are designed to provide a range of harvest alternatives that are likely to bracket the final TAC for 2004, are as follow (“ $\max F_{\text{ABC}}$ ” refers to the maximum permissible value of  $F_{\text{ABC}}$  under Amendment 56):

*Scenario 1:* In all future years,  $F$  is set equal to  $\max F_{\text{ABC}}$ . (Rationale: Historically, TAC has been constrained by ABC, so this scenario provides a likely upper limit on future TACs.)

*Scenario 2:* In all future years,  $F$  is set equal to a constant fraction of  $\max F_{\text{ABC}}$ , where this fraction is equal to the ratio of the  $F_{\text{ABC}}$  value for 2004 recommended in the assessment to the  $\max F_{\text{ABC}}$  for 2004. (Rationale: When  $F_{\text{ABC}}$  is set at a value below  $\max F_{\text{ABC}}$ , it is often set at the value recommended in the stock assessment.)

*Scenario 3:* In all future years,  $F$  is set equal to 50% of  $\max F_{\text{ABC}}$ . (Rationale: This scenario provides a likely lower bound on  $F_{\text{ABC}}$  that still allows future harvest rates to be adjusted downward when stocks fall below reference levels.)

*Scenario 4:* In all future years,  $F$  is set equal to the 1995-1999 average  $F$ . (Rationale: For some stocks, TAC can be well below ABC, and recent average  $F$  may provide a better indicator of  $F_{TAC}$  than  $F_{ABC}$ .)

*Scenario 5:* In all future years,  $F$  is set equal to zero. (Rationale: In extreme cases, TAC may be set at a level close to zero.)

Two other scenarios are needed to satisfy the MSFCMA's requirement to determine whether a stock is currently in an overfished condition or is approaching an overfished condition. These two scenarios are as follow (for Tier 3 stocks, the MSY level is defined as  $B_{35\%}$ ):

*Scenario 6:* In all future years,  $F$  is set equal to  $F_{OFL}$ . (Rationale: This scenario determines whether a stock is overfished. If the stock is expected to be above  $\frac{1}{2}$  of its MSY level in 2004 and above its MSY level in 2014 under this scenario, then the stock is not overfished.)

*Scenario 7:* In 2004 and 2005,  $F$  is set equal to  $\max F_{ABC}$ , and in all subsequent years,  $F$  is set equal to  $F_{OFL}$ . (Rationale: This scenario determines whether a stock is approaching an overfished condition. If the stock is expected to be above its MSY level in 2016 under this scenario, then the stock is not approaching an overfished condition.)

Projected catch and abundance were estimated using  $F_{40\%}$ ,  $F$  equal to the average  $F$  from 1995 to 1999,  $F$  equal to one half  $F_{40\%}$ , and  $F=0$  from 2004 to 2008 (Table 6.4). Under scenario 6 above, the year 2004 female spawning biomass is 109,976 t and the year 2014 spawning biomass is 43,639 t, above the  $B_{35\%}$  level of 41,734 t. For scenario 7 above, the year 2016 spawning biomass is 43,577 t also above  $B_{35\%}$ .

### Acceptable biological catch

ABC for 2003 using  $F_{40\%} = 0.47$  was estimated at 51,721 t. In last year's assessment ABC for 2003 using  $F_{40\%} = 0.417$  was estimated at 41,402 t (Turnock, et al. 2002).

The ABC by management area using  $F_{40\%}$  was estimated by calculating the fraction of the 2003 survey biomass in each area and applying that fraction to the ABC:

Flathead sole ABC (t) by INPFC area

|          | Western | Central | West Yakutat | East Yakutat/SE | Total  |
|----------|---------|---------|--------------|-----------------|--------|
| ABC 2004 | 13,411  | 34,433  | 3,431        | 447             | 51,721 |

### Overfishing definition

Yield at  $F_{35\%} = 0.63$  was estimated at 64,750 t.

## Summary

Table 6.5 shows a summary of model results.

## Literature cited

- Fournier, D.A. and C.P. Archibald. 1982. A general theory for analyzing catch-at-age data. Can. J.Fish.Aquat.Sci. 39:1195-1207.
- Greiwan, A. and G.F. Corliss(eds). 1991. Automatic differentiation of algorithms: theory, implementation and application. Proceedings of the SIAM Workshop on the Automatic

Differentiation of Algorithms, held Jan. 6-8, Breckenridge, CO. Soc. Indust. And Applied Mathematics, Philadelphia.

Methot, R. D. 1990. Synthesis model: An adaptable framework for analysis of diverse stock assessment data. Int. N. Pac. Fish. Comm. Bull. 50:259-277.

Press, W.H., S.A. Teukolsky, W.T.Vetterling, B.P. Flannery. 1992. Numerical Recipes in C. Second Ed. Cambridge Univ. Press. 994 p.

## Tables

Table 6.1. Catch of flathead sole in the Gulf of Alaska from 1978 to 2 October 2003.

| Year | Catch(t) |
|------|----------|
| 1978 | 451      |
| 1979 | 165      |
| 1980 | 2,068    |
| 1981 | 1,070    |
| 1982 | 1,368    |
| 1983 | 1,079    |
| 1984 | 548      |
| 1985 | 319      |
| 1986 | 147      |
| 1987 | 150      |
| 1988 | 520      |
| 1989 | 747      |
| 1990 | 1,447    |
| 1991 | 1,717    |
| 1992 | 2,034    |
| 1993 | 2,366    |
| 1994 | 2,580    |
| 1995 | 2,181    |
| 1996 | 3,107    |
| 1997 | 2,446    |
| 1998 | 1,742    |
| 1999 | 900      |
| 2000 | 1,547    |
| 2001 | 1,911    |
| 2002 | 2,145    |
| 2003 | 1,969    |

Table 6.2. Biomass estimates and standard errors from NMFS bottom trawl surveys. Note that in 2001 the eastern GOA was not surveyed. The estimated eastern survey biomass based on previous surveys was added to the 2001 biomass (153,747 t), which increased the total by about 11%.

| Survey year | Biomass(t) | Standard deviation |
|-------------|------------|--------------------|
| 1984        | 249,335    | 30,421             |
| 1987        | 179,820    | 19,227             |
| 1990        | 243,067    | 28,879             |
| 1993        | 188,592    | 24,458             |
| 1996        | 205,485    | 18,429             |
| 1999        | 207,520    | 24,418             |
| 2001        | 170,915    | 20,510             |
| 2003        | 258,609    | 20,093             |

Table 6.3. Estimated age 3+ population biomass(t), female spawning biomass(t) and age 3 recruits(1,000's).

| Year | age 3+<br>biomass | Female spawning<br>biomass | Age 3 recruits<br>(1,000's) |
|------|-------------------|----------------------------|-----------------------------|
| 1984 | 256,569           | 78,499                     | 139,157                     |
| 1985 | 265,449           | 92,933                     | 240,755                     |
| 1986 | 272,262           | 103,088                    | 250,217                     |
| 1987 | 274,447           | 108,716                    | 162,231                     |
| 1988 | 277,953           | 110,903                    | 274,518                     |
| 1989 | 281,551           | 110,936                    | 276,882                     |
| 1990 | 284,702           | 110,489                    | 251,112                     |
| 1991 | 287,181           | 109,970                    | 249,262                     |
| 1992 | 291,679           | 109,656                    | 308,048                     |
| 1993 | 291,703           | 109,878                    | 164,793                     |
| 1994 | 292,700           | 110,802                    | 260,783                     |
| 1995 | 297,317           | 112,068                    | 355,778                     |
| 1996 | 298,861           | 113,536                    | 199,319                     |
| 1997 | 295,800           | 114,026                    | 148,701                     |
| 1998 | 292,356           | 114,351                    | 197,742                     |
| 1999 | 287,703           | 115,080                    | 181,252                     |
| 2000 | 287,035           | 116,473                    | 294,309                     |
| 2001 | 287,383           | 116,325                    | 285,496                     |
| 2002 | 290,590           | 114,330                    | 317,869                     |
| 2003 | 291,421           | 111,660                    | 200,987                     |

Table 6.4. Projected female spawning biomass and yield from 2003 to 2007.

| Year           | Female spawning biomass(t) | Yield(t) |
|----------------|----------------------------|----------|
| F=F40%         |                            |          |
| 2004           | 109,976                    | 51,721   |
| 2005           | 82,401                     | 36,247   |
| 2006           | 68,613                     | 27,804   |
| 2007           | 61,771                     | 23,643   |
| 2008           | 57,414                     | 21,610   |
| F=0.016(avg F) |                            |          |
| 2004           | 109,976                    | 2,085    |
| 2005           | 110,083                    | 2,048    |
| 2006           | 111,733                    | 2,030    |
| 2007           | 113,631                    | 2,038    |
| 2008           | 114,640                    | 2,065    |
| F=0.5 F40%     |                            |          |
| 2004           | 109,976                    | 28,310   |
| 2005           | 95,367                     | 23,520   |
| 2006           | 86,878                     | 20,323   |
| 2007           | 81,770                     | 18,426   |
| 2008           | 77,766                     | 17,340   |
| F=0            |                            |          |
| 2004           | 109,976                    | 0        |
| 2005           | 111,259                    | 0        |
| 2006           | 113,915                    | 0        |
| 2007           | 116,674                    | 0        |
| 2008           | 118,429                    | 0        |



Table 6.5. Summary of results of flathead sole assessment in the Gulf of Alaska.

|  |                       |
|--|-----------------------|
| Natural Mortality                          | 0.2 females and males |
| Age of full(95%) fishery selection         | 13 females, 14 males  |
| <b>Reference fishing mortalities</b>       |                       |
| F40%                                       | 0.47                  |
| F35%                                       | 0.63                  |
| Biomass at MSY                             | N/A                   |
| Equilibrium unfished Spawning biomass (B0) | 119,240 t             |
| B35% Spawning biomass fishing at F35%      | 41,734 t              |
| B40% Spawning biomass fishing at F40%      | 47,696 t              |
| <b>Projected 2004 biomass</b>              |                       |
| Total(age 3+)                              | 292,672 t             |
| Spawning                                   | 109,976 t             |
| Overfishing level for 2004                 | 64,750 t              |

## Figures

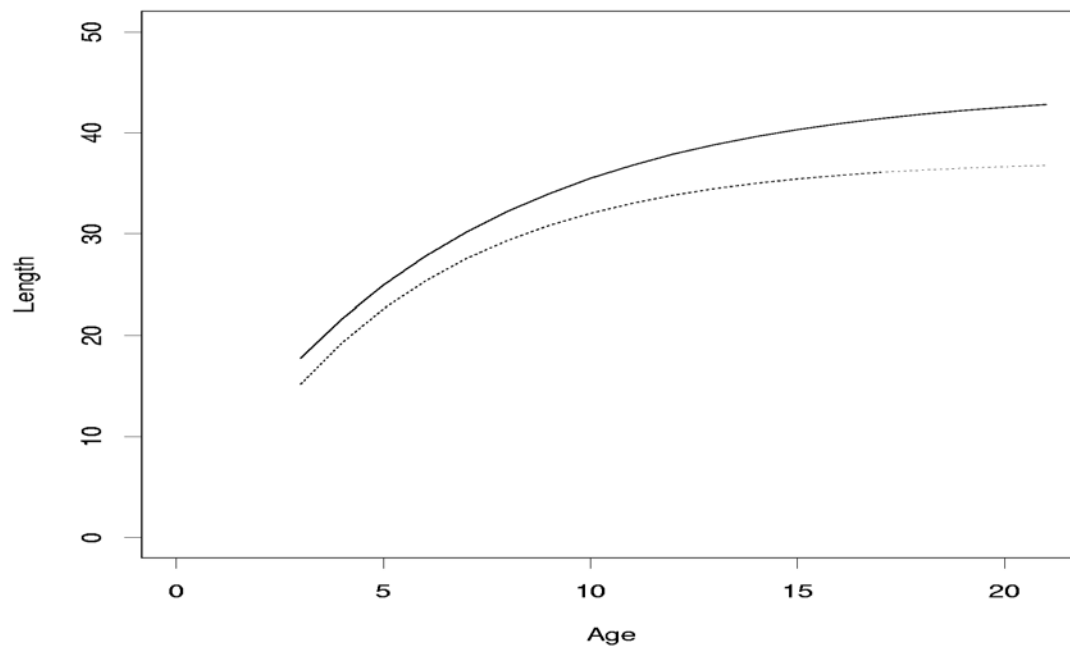


Figure 6.1. Growth for flathead sole, females solid line, males dotted line.

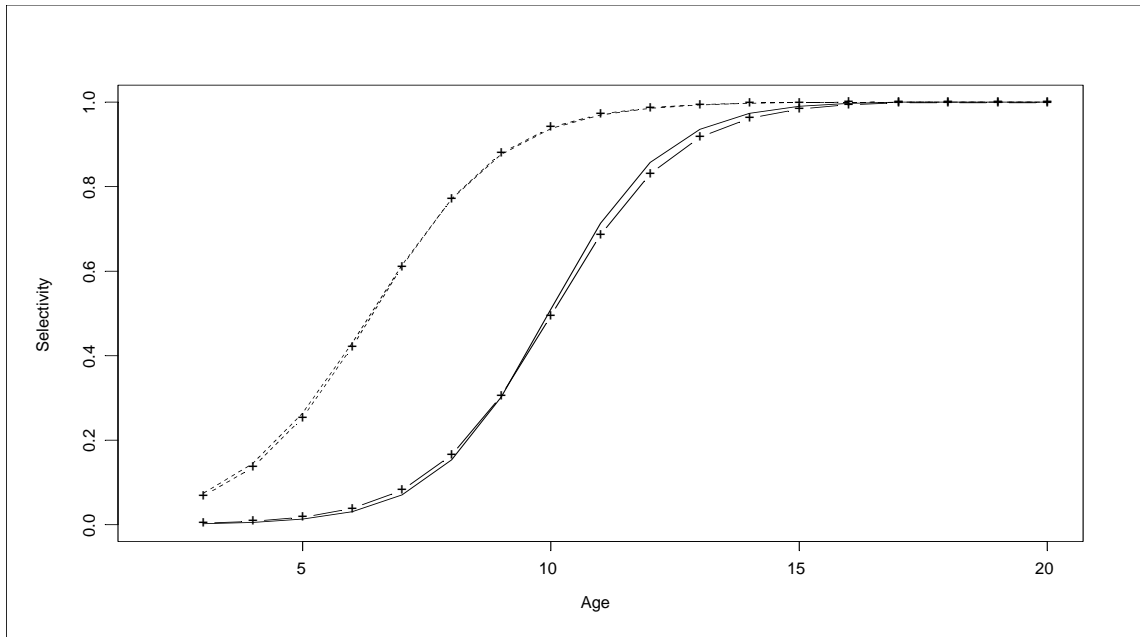


Figure 6.2. Selectivities for the survey (dotted line) and fishery (solid line). Male curve with +.

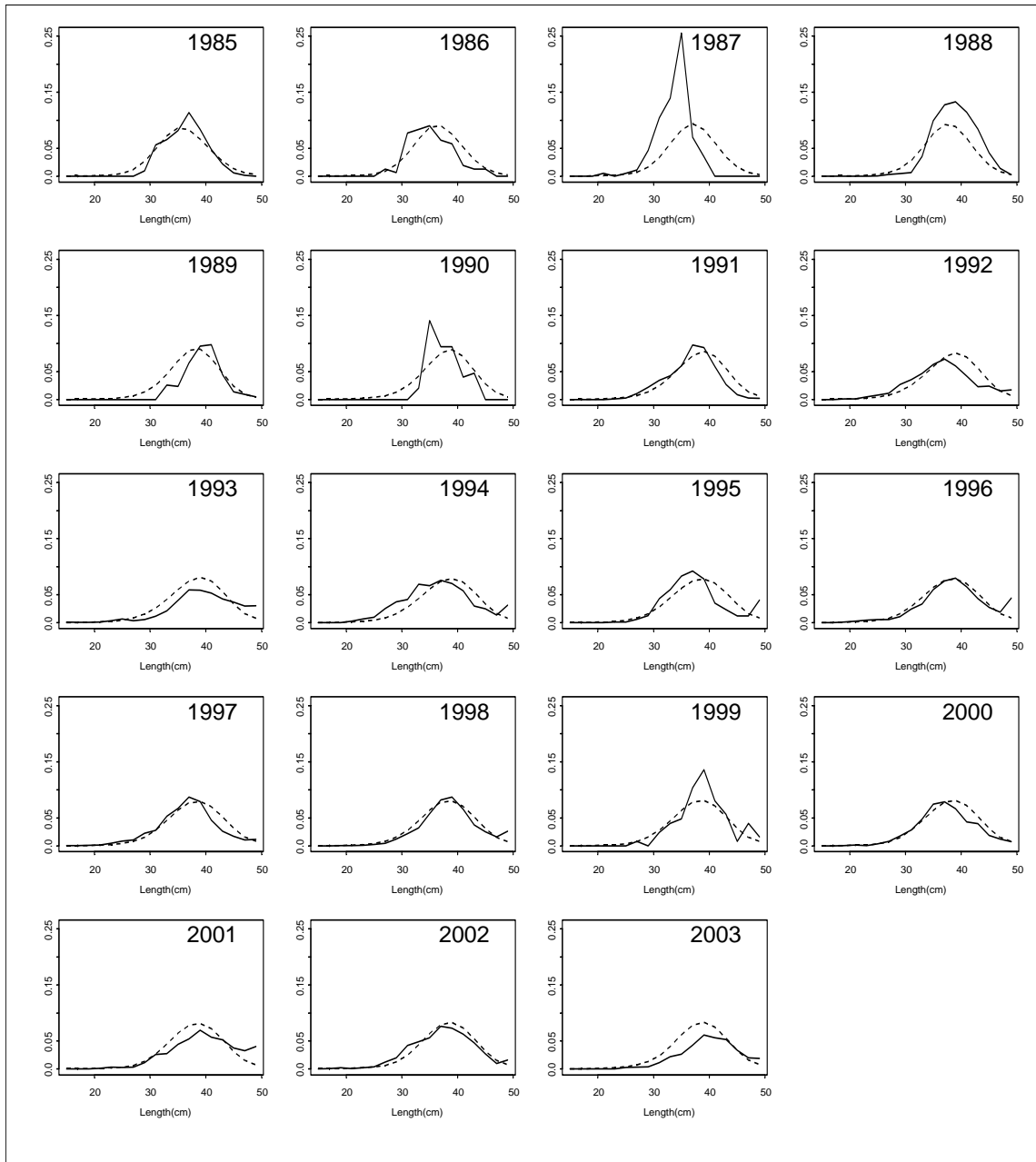


Figure 6.3. Fit to female fishery GOA flathead sole length composition data. Dashed lines represent the model prediction, solid lines represent the data.

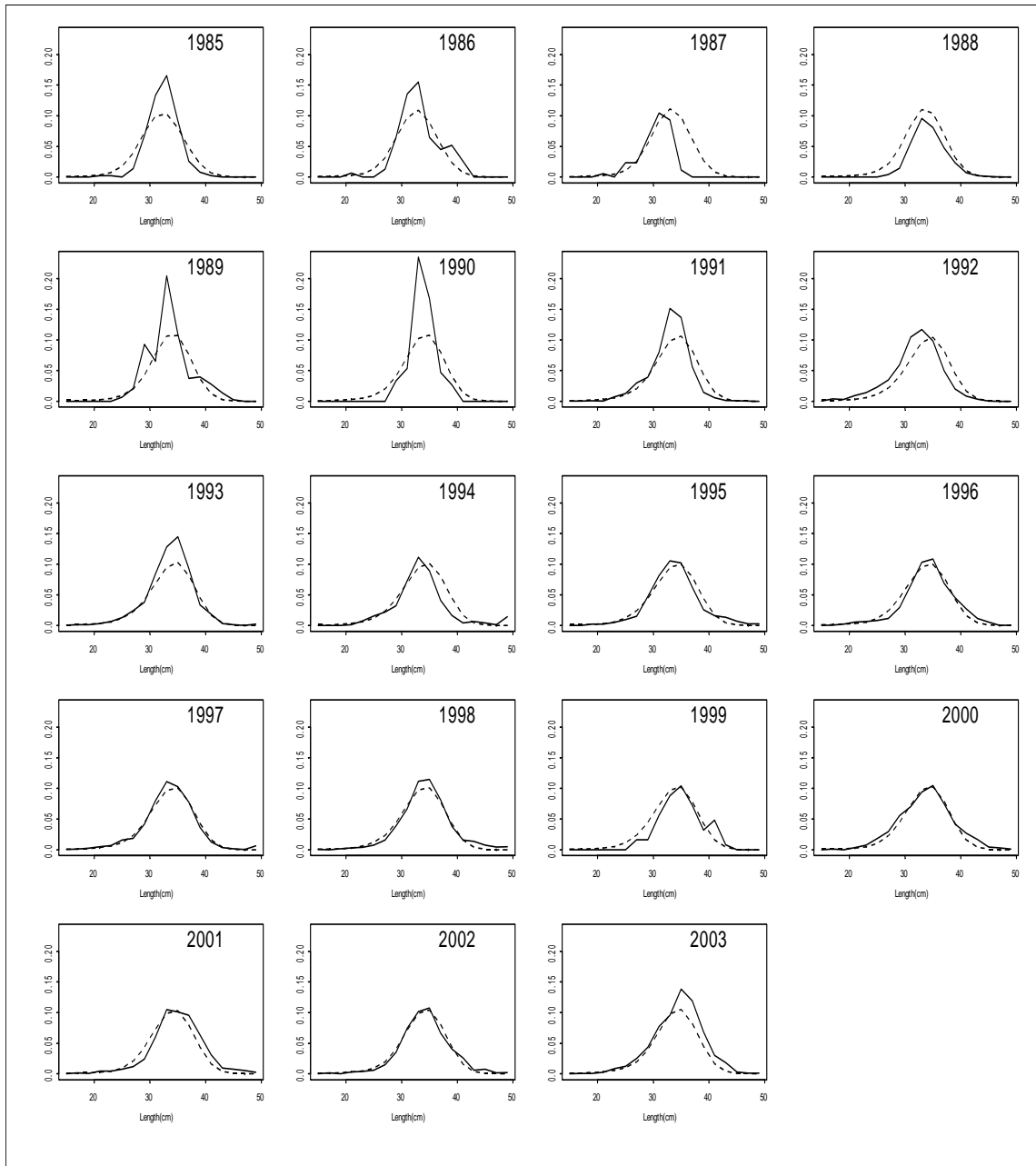


Figure 6.4. Fit to male fishery GOA flathead sole length composition data. Dashed lines represent the model prediction, solid lines represent the data.

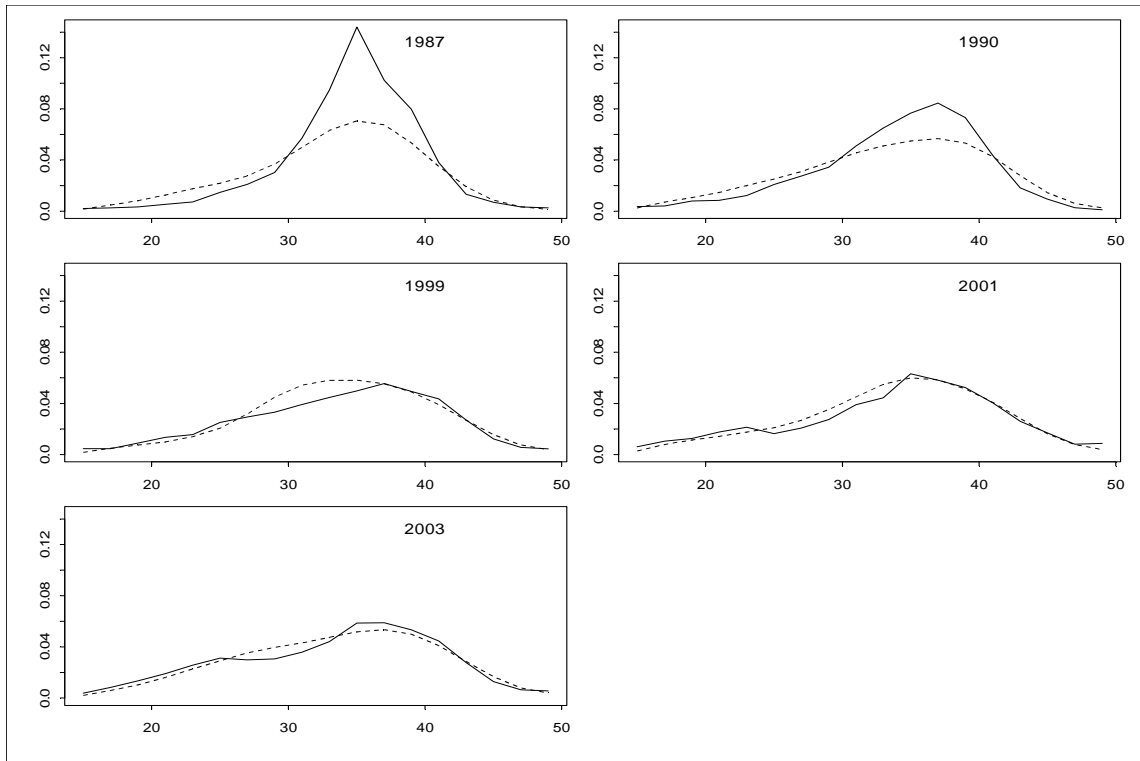


Figure 6.5. Fit to the female survey length composition data. Dashed lines represent the model prediction, solid lines represent the data.

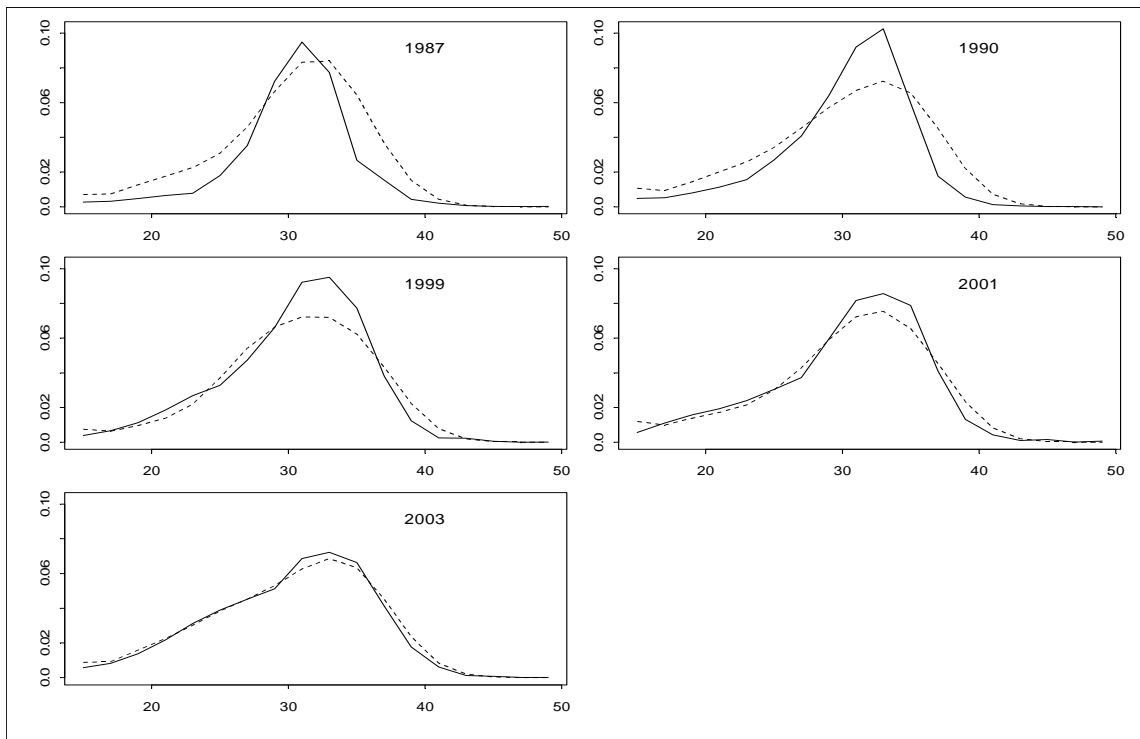


Figure 6.6. Fit to the male survey GOA flathead sole length composition data. Dashed lines represent the model prediction, solid lines represent the data.

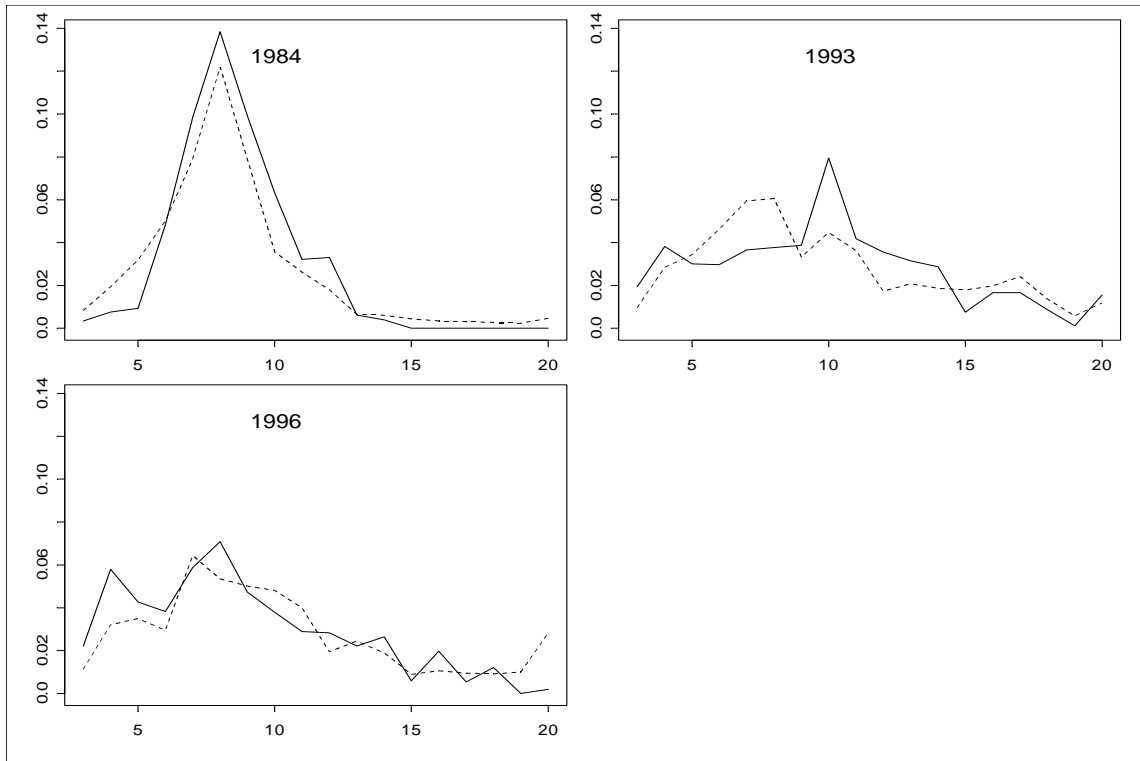


Figure 6.7. Fit to the female survey GOA flathead sole age composition data. Dashed lines represent the model prediction, solid lines represent the data.

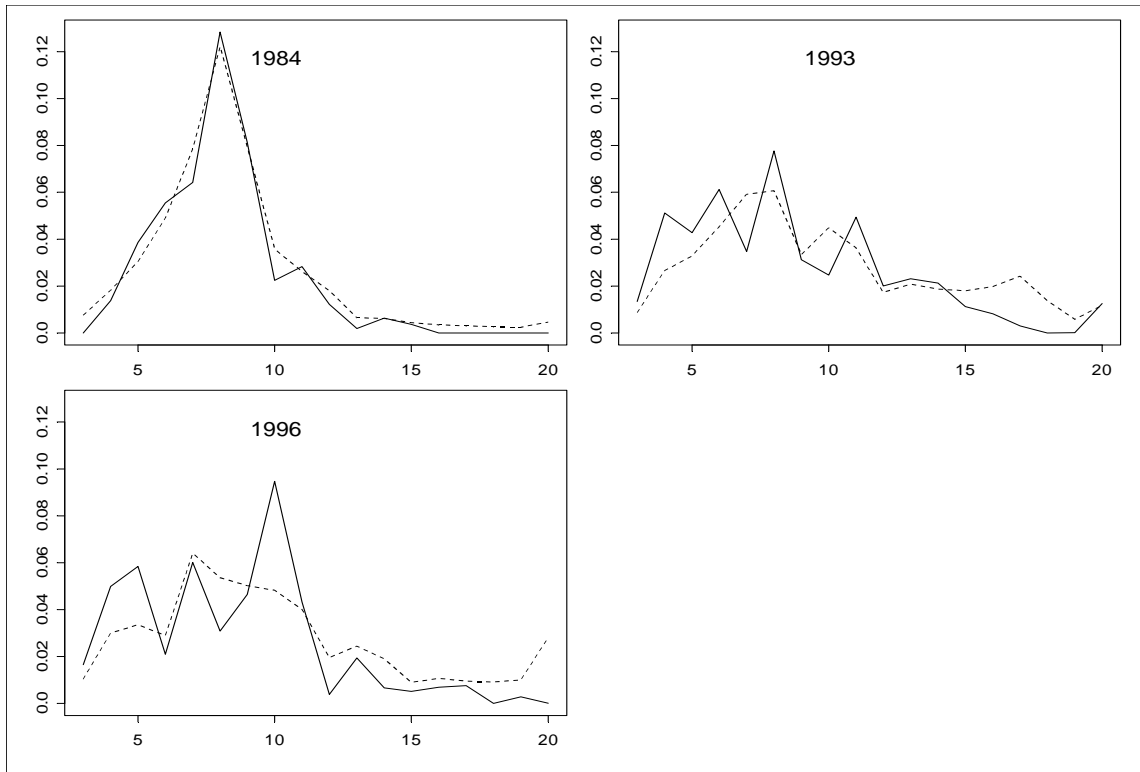


Figure 6.8. Fit to the male survey GOA flathead sole age composition data. Dashed lines represent the model prediction, solid lines represent the data.

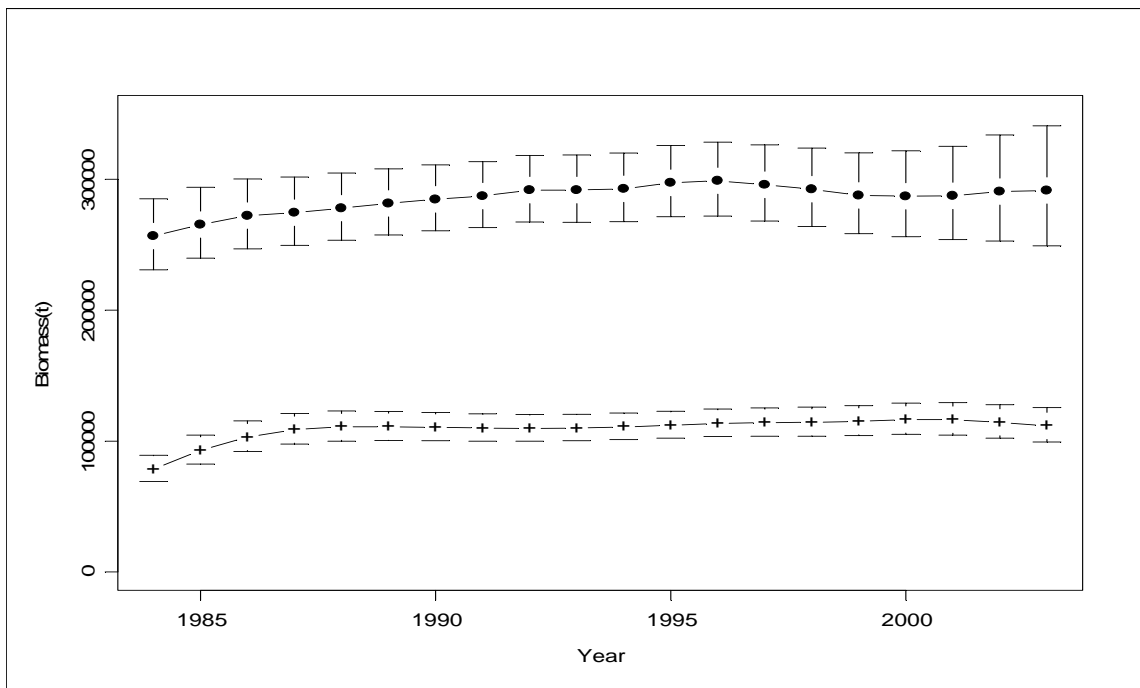


Figure 6.9. Age 3+ biomass (dotted line) and female spawning biomass (line with +) from 1984 to 2003 for GOA flathead sole. Error bars are approximate lognormal 95% confidence intervals.

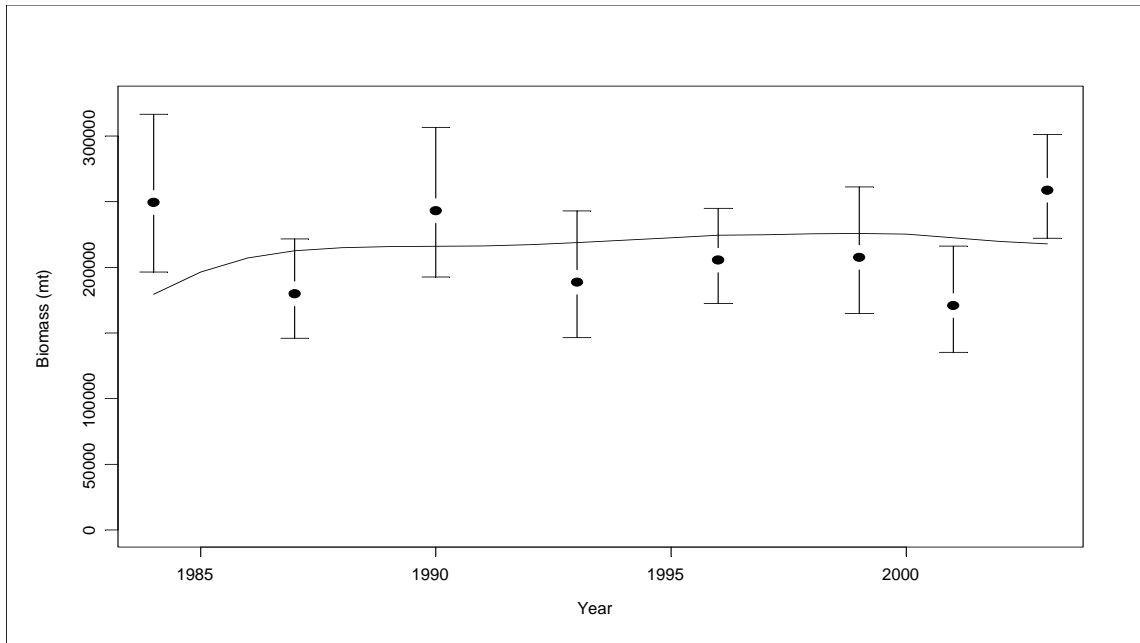


Figure 6.10. Model fit to the survey biomass estimates for GOA flathead sole. Error bars represent approximate lognormal 95% confidence intervals based on survey sampling error.



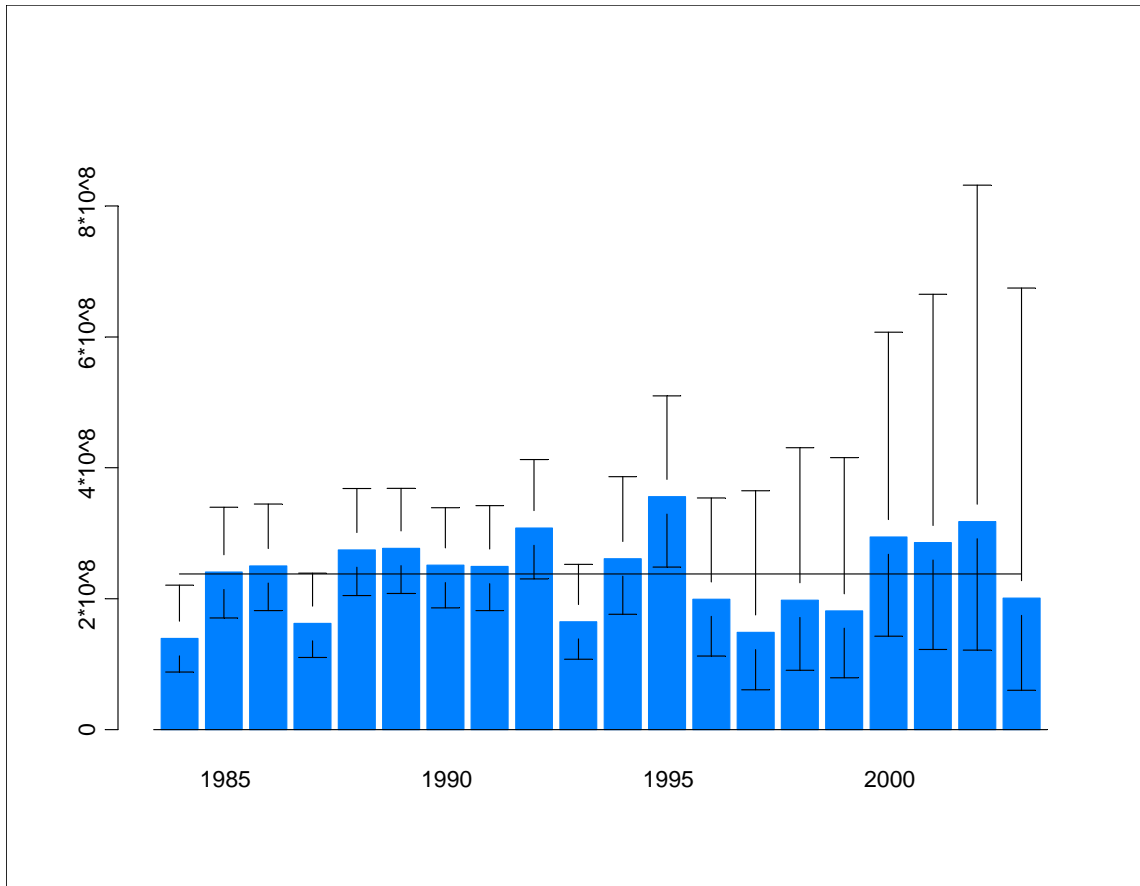


Figure 6.11. Estimated age 3 recruitments of GOA flathead sole from 1984 to 2003 with approximate 95% lognormal confidence intervals. Horizontal line is mean recruitment.

## Appendix A.

Table A.1. Model equations describing the populations dynamics.

|  |                                       |                   |   |
|--|---------------------------------------|-------------------|---|
| $N_{t,1}=R_t=R_0e^{\tau_t}$                                    | $\tau_t \sim N(0, \sigma_R^2)$        | $1 \leq t \leq T$ | Recruitment   |
| $C_{t,a} = \frac{F_{t,a}}{Z_{t,a}} (1 - e^{-Z_{t,a}}) N_{t,a}$ |                                       | $1 \leq a \leq A$ | Catch   |
| $N_{t+1,a+1} = N_{t,a} e^{-Z_{t,a}}$                           |                                       | $1 < t \leq T$    | Numbers at age  |
|  |                                       | $1 \leq a < A$    |   |
| $FSB_t = \sum_{a=1}^A w_a \phi_a N_{t,a}$                      |                                       |                   | Female spawning biomass   |
| $N_{t+1,A} = N_{t,A-1} e^{-Z_{t,A-1}} + N_{t,A} e^{-Z_{t,A}}$  |                                       | $1 < t \leq T$    | Numbers in “plus” group   |
| $Z_{t,a} = F_{t,a} + M$  |                                       |                   | Total Mortality   |
| $C_t = \sum_{a=1}^A C_{t,a}$                                   |                                       |                   | Total Catch in numbers  |
| $p_{t,a} = C_{t,a} / C$  |                                       |                   | proportion at age in the catch  |
| $Y_t = \sum_{a=1}^A w_{t,a} C_{t,a}$                           |                                       |                   | Yield   |
| $F_{t,a} = s_a E_t e^{\varepsilon_t}$                          | $\varepsilon_t \sim N(0, \sigma_R^2)$ |                   | Fishing mortality   |
| $S_a = \frac{1}{1 + e^{(-b(Age - A_{50\%}))}}$                 |                                       |                   | Selectivity- 2 parameter ascending logistic for fishery- separate for males and females |
| $S_a = \frac{1}{1 + e^{(-b(Age - A_{50\%}))}}$                 |                                       |                   | Selectivity- 2 parameter ascending logistic for survey- separate for males and females  |
| $SB_t = Q \sum_{a=1}^A w_a s_{t,a}^s N_{t,a}$                  |                                       |                   | survey biomass, $Q = 1$ .   |

Table A.2. Likelihood components.

|  |   |
|--|---|
| $\sum_{t=1}^T [\log(C_{t,obs}) - \log(C_{t,pred})]^2$  | Catch using a lognormal distribution.   |
| $\sum_{t=1}^T \sum_{a=1}^A nsamp_t * p_{obs,t,a} \log(p_{pred,t,a})$<br>- offset   | age and length compositions using a multinomial distribution. Nsamp is the observed sample size. Offset is a constant term based on the multinomial distribution. |
| offset =<br>$\sum_{t=1}^T \sum_{a=1}^A nsamp_t * p_{obs,t,a} \log(p_{obs,t,a})$  | the offset constant is calculated from the observed proportions and the sample sizes.   |
| $\sum_{t=1}^{ts} \left[ \frac{\log \left[ \frac{SB_{obs,t}}{SB_{pred,t}} \right]}{sqrt(2) * s.d.(\log(SB_{obs,t}))} \right]^2$ | survey biomass using a lognormal distribution, ts is the number of years of surveys.  |
| $\sum_{t=1}^T (\tau_t)^2$<br>$\sum_{a=3}^{15} (diff(diff(s_a)))^2$   | Recruitment, where $\tau_t \sim N(0, \sigma_R^2)$<br><br>Smooth selectivities. The sum of the squared second differences.   |

Table A.3. List of variables and their definitions used in the model.

| Variable        | Definition  |
|-----------------|---|
| T               | number of years in the model( $t=1$ is 1984 and $t=T$ is 2003)            |
| A               | number of age classes ( $A=18$ , corresponding to ages 3( $a=1$ ) to 20+) |
| $w_a$           | mean body weight(kg) of fish in age group a.                              |
| $\phi_a$        | proportion mature at age a  |
| $R_t$           | age 3( $a=1$ ) recruitment in year t                                      |
| $R_0$           | geometric mean value of age 3 recruitment                                 |
| $\tau_t$        | recruitment deviation in year t   |
| $N_{t,a}$       | number of fish age a in year t  |
| $C_{t,a}$       | catch number of age group a in year t                                     |
| $p_{t,a}$       | proportion of the total catch in year t that is in age group a            |
| $C_t$           | Total catch in year t   |
| $Y_t$           | total yield(tons) in year t   |
| $F_{t,a}$       | instantaneous fishing mortality rate for age group a in year t            |
| M               | Instantaneous natural mortality rate                                      |
| $E_t$           | average fishing mortality in year t                                       |
| $\varepsilon_t$ | deviations in fishing mortality rate in year t                            |
| $Z_{t,a}$       | Instantaneous total mortality for age group a in year t                   |
| $s_a$           | selectivity for age group a   |

Table A.4. Estimated parameters for the model. There were 68 total parameters estimated in the model.

| Parameter  | Description  |
|--|--|
| $\log(R_0)$  | log of the geometric mean value of age 3 recruitment               |
| $\tau_t$ $1984 \leq t \leq 2003$ , plus 18 parameters for the initial age composition equals 38. | Recruitment deviation in year t                                    |
| $\log(f_0)$  | log of the geometric mean value of fishing mortality               |
| $\varepsilon_t$ $1984 \leq t \leq 2003$ , 20 parameters  | deviations in fishing mortality rate in year t                     |
| Slope and age at 50% selected – 4 parameters   | selectivity parameters for the fishery for males and females.      |
| Slope and age at 50% selected – 4 parameters   | selectivity parameters for the survey data, for males and females. |

Table A.5. Fixed parameters in the model.

| Parameter   | Description  |
|---|--|
| $M = 0.2$   | Natural mortality  |
| $Q = 1.0$   | Survey catchability  |
| $L_{inf}$ , $L_{age2}$ , $k$ , $cv$ of length at age 2 and age 20 for males and females | von Bertalanffy Growth parameters estimated from the 1984-1996 survey length and age data. |

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